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ON TWO-CAVITY COUPLING[2]

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Abstract

This work presents research results on a novel analytical model of electromagnetic system coupling through small size holes. The key problem regarding the coupling of two cavities through an aperture in separating screen of finite thickness without making assumption on smallness of any parameters is considered. We are the first to calculate on the base of rigorous electromagnetic approach the coupling coefficients of the cylindrical cavities within the limit of small aperture and infinitely thin separating screen. The numeric results of electromagnetic characteristic dependencies that have been impossible to perform on the base of previous models are given.

1 Introduction

The problem of electromagnetic coupling has been in the focus of scientific attention for over 40 years. The approach of tackling this problem with the use of the concepts of equivalent electric and magnetic dipole moments, suggested in [1, 2], proved to be fruitful. On its base various electromagnetic characteristics of interacting objects have been studied (see [3]-[10]) and literature cited therein). The key element of this approach is employment of the static analysis used for determination the fields in the immediate vicinity of the hole. Clearly, this procedure is valid only if the hole dimensions are small compared to the wave length. Besides, the apertures have to be placed at a remote distance from the borders of the electromagnetic systems being considered. This notwithstanding, the developed methods allowed not only to calculate the number of important characteristics, but formulate (or lay the basis) for entirely new approaches for consideration of different RF-devices. This approach exerted considerable influence on the theory of slow-wave structures based on utilization of resonant properties of electromagnetic systems (disk-loaded waveguides, coupled-cavity chains, etc.) However, even to this day, there have not been developed general methods of calculations of small aperture coupling coefficients from which the static results could be obtained by means of the limit transition $\omega a/c \rightarrow 0$. Development of such methods would permit not only to assess the region of applicability of static results, but also to expand the frontiers of problems regarding RF-interactions that can be rigorously solve (correct evaluation of the separating screen thickness, the vicinity of walls, etc.) It must be noted that several efforts[3] were made to push forward the frontier of applicability of the static approach in cavity coupling [4, 10]. However, the accurateness of the proposed techniques[4] cannot be proven within the framework of the models considered. Development of novel analytical method for investigation of electromagnetic system coupling through small-size apertures is also important considering the fact that there are difficulties of utilization the widely developed electromagnetic simulations techniques in this particular area. These difficulties are associated with the

[3]Note, that some characteristics of RF coupling can be obtained by using variation technique (see, for example, [11, 12]). But on the base of this technique it is difficult to derive the coupled equations that describe the system under consideration

[4]There are plenty of works concerning the diffraction by a circula conducting disk (or the complimentary problem for a circular aperture in an infinite plane conducting screen)(see,for example, [13]), but the cavity coupling problem includes the development of the procegure for deriving specific coupled equations

requirements of very high precision mathematical models to be used for small coupling holes, since the relative correctness of a model has to be smaller than the coupling coefficients. This paper presents research results in the development of a novel analytical model for studies on electromagnetic systems coupling through small-size apertures ([14]-[16]). Considered is the key problem of two cavity coupling through an aperture in separating screen of finite thickness without making assumption on smallness of any parameters.

2 Problem Definition

Let us consider two ideal conducting co-axial cylindrical cavities coupled through a cylindrical aperture of the radius a in the separating planar screen of the thickness t. The radii and lengths of the first and second cavities will be designated b_1, d_1 and b_2, d_2, respectively. To construct a mathematical model of the electromagnetic system under consideration, we will use a relatively novel method of partial cross-over regions (see, for instance, [17, 18]). As the first and second regions, we will take the cylindrical cavity volumes; for the third, a cylinder that is co-axial with the coupling hole , its radius being equal $b_3 = a$. This cylinder projects into the area of the first cavity for the length d_{1*} and into the second one for the length d_{2*} , the cylinder length being $l_* = d_{1*} + d_{2*} + t$.

In each region, we expand the electromagnetic fields in terms of the orthonormal complete set of field functions without the hole:

$$\vec{E} = \sum_{n,s} e_{n,s}^{(i)} \vec{\mathcal{E}}_{n,s}^{(i)} + \sum_{n,s} e_{n,s}^{\prime(i)} \vec{\mathcal{E}}_{n,s}^{\prime(i)} \tag{1}$$

$$\vec{H} = \sum_{n,s} h_{n,s}^{(i)} \vec{\mathcal{H}}_{n,s}^{(i)} + \sum_{n,s} h_{n,s}^{\prime(i)} \vec{\mathcal{H}}_{n,s}^{\prime(i)} \tag{2}$$

where $\vec{\mathcal{E}}_{n,s}^{(i)}, \vec{\mathcal{H}}_{n,s}^{(i)}$ - are solenoidal and $\vec{\mathcal{E}}_{n,s}^{\prime(i)}, \vec{\mathcal{H}}_{n,s}^{\prime(i)}$ - irrotational sub- sets. For axial-symmetric modes $e_{n,s}^{\prime(i)} = h_{n,s}^{\prime(i)} = 0$, while the set of solenoidal basic functions takes on the form:

$$\mathcal{E}_{n,s,\mathrm{z}}^{(i)} = \frac{\lambda_s^2 c}{b_i^2 \omega_{n,s}^{(i)} N_{n,s}^{(i)}} \cos(k_n^{(i)} \xi_i) J_0\left(\lambda_s r / b_i\right), \tag{3}$$

$$\mathcal{E}_{n,s,\mathrm{r}}^{(i)} = \frac{\lambda_s k_n^{(i)} c}{b_i \omega_{n,s}^{(i)} N_{n,s}^{(i)}} \sin(k_n^{(i)} \xi_i) J_1\left(\lambda_s r / b_i\right), \tag{4}$$

$$\mathcal{H}_{n,s,\phi}^{(i)} = -i \frac{\lambda_s}{b_i N_{n,s}^{(i)}} \cos(k_n^{(i)} \xi_i) J_1\left(\lambda_s r / b_i\right), \tag{5}$$

where

$$i = 1, 2, 3;\; s = 1, 2 \ldots \infty;\; n = 0, 1 \ldots \infty; J_0\left(\lambda_s\right) = 0;$$

$$\omega_{n,s}^{(i)} = c\sqrt{k_n^{(i)2} + \lambda_s^2 / b_i^2};\; k_n^{(i)} = \pi n / d_i;\; N_{n,s}^{(i)} = \sqrt{\pi \theta_n d_i \lambda_s^2 J_1^2(\lambda_s)/2};$$

$$\theta_n = \begin{cases} 2, & n = 0, \\ 1, & n \neq 0, \end{cases} \quad \xi_1 = z;\; \xi_2 = z - d_1 - t;\; \xi_3 = z - d_1 - d_{1*}.$$

The basic set (3-5) satisfies the orthonormality condition:

$$\int_v \vec{\mathcal{E}}_{n,s}^{(i)} \vec{\mathcal{E}}_{n',s'}^{(i)*} dV = \int_v \vec{\mathcal{H}}_{n,s}^{(i)} \vec{\mathcal{H}}_{n',s'}^{(i)*} dV = \delta_{n,n'} \delta_{s,s'}. \tag{6}$$

Coefficients $e^{(i)}_{n,s}$ in the expansion (1-2) are determined by the electric field tangential components at the boundaries of the chosen regions

$$\left(\omega^{(i)2}_{n,s} - \omega^2\right) e^{(i)}_{n,s} = -ic\omega^{(i)}_{n,s} \int_S \left[\vec{E}\vec{\mathcal{H}}^{*(i)}_{n,s}\right] d\vec{s}. \tag{7}$$

Since the electric field tangential component $\vec{E_\tau}$ on a metallic surface is zero, then, in Eq.(7) the integration surfaces for the first and second regions will be circles located on the opposite planes of the hole in the screen, while for the third one, two cylindrical surfaces and two circles, following which this particular region is in contact with the former two regions. Remembering this, we derive from (7) the following:

$$\left(\omega^{(i)2}_{k,l} - \omega^2\right) e^{(i)}_{k,l} = \sum_{n,s} e^{(3)}_{n,s} L^{(i)}_{n,s,k,l}, \quad i = 1, 2, \tag{8}$$

$$e^{(3)}_{n,s} = \sum_{n',s'} \left(e^{(1)}_{n',s'} T^{(1)}_{n',s',n,s} + e^{(2)}_{n',s'} T^{(2)}_{n',s',n,s}\right), \tag{9}$$

$$L^{(1)}_{n,s,k,l} = ic\omega^{(1)}_{k,l} 2\pi \int_0^a rdr \left(\mathcal{E}^{(3)}_{n,s,\mathrm{r}} \mathcal{H}^{*(1)}_{k,l,\phi}\right)_{z=d_1},$$

$$L^{(2)}_{n,s,k,l} = -ic\omega^{(2)}_{k,l} 2\pi \int_0^a rdr \left(\mathcal{E}^{(3)}_{n,s,\mathrm{r}} \mathcal{H}^{*(2)}_{k,l,\phi}\right)_{z=d_1+t},$$

$$T^{(1)}_{n',s',n,s} = \frac{2\pi ic\omega^{(3)}_{n,s}}{\omega^{(3)2}_{n,s} - \omega^2} \times$$

$$\times \left[-a \int_{d_1-d_{1*}}^{d_1} dz \left(\mathcal{E}^{(1)}_{n',s',\mathrm{z}} \mathcal{H}^{*(3)}_{n,s,\phi}\right)_{r=a} - \int_0^a rdr \left(\mathcal{E}^{(1)}_{n',s',\mathrm{r}} \mathcal{H}^{*(3)}_{n,s,\phi}\right)_{z=d_1-d_{1*}}\right],$$

$$T^{(2)}_{n',s',n,s} = \frac{2\pi ic\omega^{(3)}_{n,s}}{\omega^{(3)2}_{n,s} - \omega^2} \times$$

$$\times \left[-a \int_{d_1+t}^{d_1+t+d_{2*}} dz \left(\mathcal{E}^{(2)}_{n',s',\mathrm{z}} \mathcal{H}^{*(3)}_{n,s,\phi}\right)_{r=a} - \int_0^a rdr \left(\mathcal{E}^{(2)}_{n',s',\mathrm{r}} \mathcal{H}^{*(3)}_{n,s,\phi}\right)_{z=d_1+t+d_{2*}}\right].$$

Substituting (9) into (8), and introducing for the sake of convenience new variables

$$a^{(i)}_{k,l} = e^{(i)}_{k,l} \frac{\lambda_l J_0\left(\lambda_l a/b_i\right)}{\omega^{(i)}_{k,l} \sqrt{\theta_k} J_1(\lambda_l)}$$

instead $e^{(i)}_{k,l}$ $(i = 1, 2)$, we get a set of equations for field amplitudes only in the 1-st and 2-nd regions

$$\theta_k Z^{(i)}_{k,l} a^{(i)}_{k,l} = \sum_{n',s'} \left(a^{(1)}_{n',s'} V^{(i,1)}_{n',s',k,l} + a^{(2)}_{n',s'} V^{(i,2)}_{n',s',k,l}\right), \quad i = 1, 2 \tag{10}$$

where $Z^{(i)}_{k,l} = \omega^{(i)2}_{k,l} - \omega^2$,

$$V^{(i,j)}_{n',s',k,l} = \sum_{n',s'} \frac{\lambda_l J_1\left(\lambda_{s'}\right) J_0\left(\lambda_l a/b_i\right) \omega^{(i)}_{n',s'}}{\lambda_{s'} J_1\left(\lambda_l\right) J_0\left(\lambda_{s'} a/b_i\right) \omega^{(i)}_{n,s}} \sqrt{\theta_{n'}\theta_k} T^{(j)}_{n',s',n,s} L^{(i)}_{n',s',k,l}.$$

After making simple, although cumbersome, calculations we obtain the following expression for the coefficients $V^{(i,j)}_{n',s',k,l}$:

$$V^{(i,j)}_{n',s',k,l} = (-1)^{1+i(1+k)+j(1+n')} \alpha_{i,j} \gamma_{l,i} \sum_{s} \sigma_{s,l,i} \Delta_{s,n',j} \times$$

$$\times \left[f^{(i,j)}_s - \beta_i Z^{(j)}_{n',s'} \sigma_{s,s',j} R_{n',j} F^{(i,j)}_s \right], \tag{11}$$

where

$$\alpha_{i,j} = 4a^3c^2 \left(b_i^2 b_j^2 \sqrt{d_1 d_2} \right)^{-1}, \gamma_{l,i} = \lambda_l^2 J_0^2 \left(\lambda_l a / b_i \right) / J_1(\lambda_l),$$

$$\sigma_{s,l,i} = \left(\lambda_s^2 - a^2 \lambda_l^2 / b_i^2 \right)^{-1}, \Delta_{s,n,j} = \left[\lambda_s^2 - \Omega^2 + (\pi a n / d_j)^2 \right]^{-1},$$

$$\beta_i = 2a^3 / \left(c^2 d_i \right), R_{n,j} = \pi n \sin(\pi n d_{j*} / d_j),$$

$$F^{(i,j)}_s = \frac{1}{\sinh(q_s)} \begin{cases} \sinh[q_s \left(1 - d_{i*}/l_*\right)], & i = j, \\ \sinh[q_s d_{i*}/l_*], & i \neq j, \end{cases}$$

$$f^{(i,j)}_s = \frac{\mu_s}{\sinh(q_s)} \times$$

$$\times \begin{cases} \cosh[q_s] - \cosh[q_s \left(1 - 2d_{i*}/l_*\right)], & i = j, \\ \cosh[q_s \left(d_{2*} + d_{1*}\right)/l_{*)}] - \cosh[q_s \left(d_{2*} - d_{1*}\right)/l_{*)}], & i \neq j, \end{cases}$$

$$q_s = \mu_s l_* / a, \ \mu_s = \sqrt{\lambda_s^2 - \Omega^2}, \ \Omega = \omega a / c.$$

The uniform set of Eqs.(10) describes the interaction of two infinite sets of oscillators, which are eigenmodes of closed cavities (without the coupling hole in the separating screen), being, in principle, fit to be used for calculations of necessary electromagnetic characteristics of coupled cavities. However, the set of Eqs.(10) has three drawbacks that make it difficult to carry out both analytical investigations and numerical calculations. Firstly, the structure of this set of equations does not yield a possibility to obtain analytical results, in particular, in the well studded limit $t = 0$ and $a \to 0$. Secondly, this set is two-dimensional, and it is necessary to have great calculative resources to solve it. Thirdly, owing to the presence of field singularity peculiarities at acute angles of the hole in the screen the coefficients $V^{(i,j)}_{n',s',k,l}$ decrease slowly with increasing indices. Our studies show that the set of Eqs.(10) can be reduced to such a form that has no first or second drawbacks. Below are the results of these studies.

3 Derivation of Basic Equations

Let us seek for the amplitudes of eigenmodes $a^{(i)}_{k,l}$, except for the fundamental modes $((k,l) \neq (0,1))$, in the form:

$$\theta_k Z^{(i)}_{k,l} a^{(i)}_{k,l} = a^{(1)}_{0,1} x^{(i,1)}_{k,l} + a^{(2)}_{0,1} x^{(i,2)}_{k,l}. \tag{12}$$

By introducing two new sequences of unknown values $\left\{x_{k,l}^{(i,1)}\right\}$ and $\left\{x_{k,l}^{(i,2)}\right\}$, instead of one $\left\{a_{k,l}^{(i)}\right\}$, we can impose one additional condition on these new sequences. Let us assume that $\left\{x_{k,l}^{(i,1)}\right\}$ satisfies the equations

$$x_{k,l}^{(i,1)} = {\sum_{n',s'}}' \left(\frac{x_{n',s'}^{(1,1)}}{\theta_{n'} Z_{n',s'}^{(1)}} V_{n',s',k,l}^{(i,1)} + \frac{x_{n',s'}^{(2,1)}}{\theta_{n'} Z_{n',s'}^{(2)}} V_{n',s',k,l}^{(i,2)} \right) + V_{0,1,k,l}^{(i,1)}, \tag{13}$$

where $(k,l) \neq (0,1)$, $i = 1,2$.

Then from Eqs.(10) it follows that $\left\{x_{k,l}^{(i,2)}\right\}$ ($(k,l) \neq (0,1)$, $i = 1,2$) must satisfy the relationships

$$x_{k,l}^{(i,2)} = {\sum_{n',s'}}' \left(\frac{x_{n',s'}^{(1,2)}}{\theta_{n'} Z_{n',s'}^{(1)}} V_{n',s',k,l}^{(i,1)} + \frac{x_{n',s'}^{(2,2)}}{\theta_{n'} Z_{n',s'}^{(2)}} V_{n',s',k,l}^{(i,2)} \right) + V_{0,1,k,l}^{(i,2)}, \tag{14}$$

In Eqs.(13,14) and elsewhere below the prime in sums indicate that $(n',s') \neq (0,1)$. It follows from Eq.(10) that the amplitudes of fundamental modes $(k,l) = (0,1)$ should satisfy the equations

$$\begin{aligned} 2Z_{0,1}^{(i)} a_{0,1}^{(i)} = a_{0,1}^{(1)} &\left[{\sum_{n',s'}}' \left(\frac{x_{n',s'}^{(1,1)}}{\theta_{n'} Z_{n',s'}^{(1)}} V_{n',s',k,l}^{(i,1)} + \frac{x_{n',s'}^{(2,1)}}{\theta_{n'} Z_{n',s'}^{(2)}} V_{n',s',k,l}^{(i,2)} \right) + V_{0,1,0,1}^{(i,1)} \right] + \\ &+ a_{0,1}^{(2)} \left[{\sum_{n',s'}}' \left(\frac{x_{n',s'}^{(1,2)}}{\theta_{n'} Z_{n',s'}^{(1)}} V_{n',s',k,l}^{(i,1)} + \frac{x_{n',s'}^{(2,2)}}{\theta_{n'} Z_{n',s'}^{(2)}} V_{n',s',k,l}^{(i,2)} \right) + V_{0,1,0,1}^{(i,2)} \right]. \end{aligned} \tag{15}$$

Let us denote

$$\begin{aligned} w_s^{(i,j)} &= (-1)^{j+1} 3\pi \times \\ &\times \left(f_s^{(i,1)} y_s^{(1,j)} - f_s^{(i,2)} y_s^{(2,j)} - \beta_1 F_s^{(i,1)} p_s^{(1,j)} + \beta_2 F_s^{(i,2)} p_s^{(2,j)} \right), \end{aligned} \tag{16}$$

where

$$y_s^{(i,j)} = \frac{\alpha_{i,i}}{\alpha_{i,j}} \left[\delta_{i,j} \Delta_{s,0.i} + {\sum_{n',s'}}' (-1)^{i \times n'} \Delta_{s,n',i} \frac{x_{n',s'}^{(i,j)}}{\theta_{n'} Z_{n',s'}^{(i)}} \right],$$

$$p_s^{(i,j)} = \frac{\alpha_{i,i}}{\alpha_{i,j}} {\sum_{n',s'}}' (-1)^{i \times n'} \Delta_{s,n',i} \sigma_{s,s',i} R_{n',s} \frac{x_{n',s'}^{(i,j)}}{\theta_{n'}}$$

Then, Eqs.(15) can be reduce to the form

$$\left(\omega_{0,1}^{(1)2} - \omega^2 \right) a_{0,1}^{(1)} = -\omega_{0,1}^{(1)} \frac{2}{3\pi J_1^2(\lambda_1)} \frac{a^3}{b_1^2 d_1} \left[a_{0,1}^{(1)} \Lambda_{1,1} - \frac{b_1^2 \sqrt{d_1}}{b_2^2 \sqrt{d_2}} a_{0,1}^{(2)} \Lambda_{1,2} \right] \tag{17}$$

$$\left(\omega_{0,1}^{(2)2} - \omega^2 \right) a_{0,1}^{(2)} = -\omega_{0,1}^{(2)} \frac{2}{3\pi J_1^2(\lambda_1)} \frac{a^3}{b_2^2 d_2} \left[a_{0,1}^{(2)} \Lambda_{2,2} - \frac{b_2^2 \sqrt{d_2}}{b_2^2 \sqrt{d_2}} a_{0,1}^{(1)} \Lambda_{2,1} \right] \tag{18}$$

where the coefficients $\Lambda_{i,k}$, which define the frequency shifts and cavities coupling, are determined by the expression

$$\Lambda_{i,k} = J_0 \left(\lambda_1 a / b_i \right) \sum_{s=1}^{\infty} \sigma_{s,1,i} w_s^{(i,k)} \tag{19}$$

and $w_s^{(i,k)}$ are the solutions of the following pair of sets of linear algebraic equations

$$\left\{\begin{array}{l} w_m^{(1,1)} + \sum\limits_{s=1}^{\infty} \left(w_s^{(1,1)} G_{m,s}^{(1,1)} + w_s^{(2,1)} G_{m,s}^{(1,2)} \right) = 3\pi f_m^{(1,1)} / \mu_m^2, \\ w_m^{(2,1)} + \sum\limits_{s=1}^{\infty} \left(w_s^{(2,1)} G_{m,s}^{(2,2)} + w_s^{(2,1)} G_{m,s}^{(2,1)} \right) = 3\pi f_m^{(2,1)} / \mu_m^2, \end{array}\right. \tag{20}$$

$$\left\{\begin{array}{l} w_m^{(2,2)} + \sum\limits_{s=1}^{\infty} \left(w_s^{(2,2)} G_{m,s}^{(2,2)} + w_s^{(1,2)} G_{m,s}^{(2,1)} \right) = 3\pi f_m^{(2,2)} / \mu_m^2, \\ w_m^{(1,2)} + \sum\limits_{s=1}^{\infty} \left(w_s^{(1,2)} G_{m,s}^{(1,1)} + w_s^{(2,2)} G_{m,s}^{(1,2)} \right) = 3\pi f_m^{(1,2)} / \mu_m^2, \end{array}\right. \tag{21}$$

where

$$G_{m,s}^{(i,j)} = f_m^{(i,j)} T_{m,s}^{(j)} - F_m^{(i,j)} \delta_{m,s} \frac{\sinh[\mu_m (d_i - d_{i*})/a]}{\sinh[\mu_m d_i / a]},$$

$$T_{m,s}^{(j)} = \pi \frac{a}{b} \sum_{s=1}^{\infty} \frac{\theta_l^{(j)3} J_0^2(\theta_l^{(j)}) E_l\left(a/d_j, \nu_l^{(j)}\right)}{\chi_l \left(\lambda_m^2 - \theta_l^{(j)2}\right)\left(\lambda_s^2 - \theta_l^{(j)2}\right)} - \frac{1}{2}\delta_{m,s} E_2\left(a/d_j, \mu_m\right) +$$

$$+ \frac{\pi a^2}{\mu_m^2 b_j d_j} \frac{\theta_1^{(j)3} J_0^2\left(\theta_1^{(j)}\right)}{\left(\lambda_m^2 - \theta_1^{(j)2}\right)\left(\lambda_s^2 - \theta_1^{(j)2}\right)},$$

$$E_l(x,y) = \left\{\begin{array}{ll} \coth(y/x)/y - x/y^2, & l = 1, \\ \coth(y/x)/y, & l \neq 1, \end{array}\right.$$

$$\theta_l^{(j)} = a\lambda_l / b_j,\ \chi_l = \pi \lambda_l J_1^2(\lambda_l)/2,\ \nu_l^{(j)} = \sqrt{\theta_l^{(j)2} - \Omega^2}.$$

Thus, the set of Eqs.(10), describing the coupling of infinite number of oscillators (eigenmodes of closed cavities), has been rigorously reduced to such a form that formally describes the interaction of two basic oscillators. In the case considered, E_{010} modes of the closed cavities were chosen to be such basic oscillators. Yet, this choosing is arbitrary and determined as the problem requirements demand. The entire spectrum of the resonance properties of the coupled cavity system according to this approach is contained in the dependence of $\Lambda_{i,k}$ on frequency. Such form of description of the coupled cavities is convenient for solving many such problems in which electromagnetic characteristics are studied in a limited frequency range determined by the interactions of two adjacent eigenmodes.

4 Research Results

Before starting to discuss the results of analysis of Eqs.(17,18), let us dwell on the problem of choosing the geometrical dimensions of the auxiliary region 3, namely on the choice of d_{i*} values. Results of our calculations show that the solution of the linear algebraic equations obtained by the truncation of infinite systems (20,21) at appropriate S and L values do not depend on d_{i*} (we designate the maximum value of the index s in (20,21) as S and the maximum value of the index l in the sum that determine $T_{m,s}^{(j)}$ as L). Thus, for instance, at $S = 100$, $L = 40000$, $d_1 = d_2 = 3.5$ cm, $b_1 = b_2 = 4$ cm, $t = 0.4$ cm, $a = 1$ cm,

$f = 0$ and $d_* = d_{1*} = d_{2*}$ the calculations yield: $d_* = 3.5$ cm — $\Lambda_{1,1} = 0.773125$, $d_* = 10^{-7}$ cm — $\Lambda_{1,1} = 0.773125$.

Let us consider the case of small-size apertures ($a \to 0$) and an infinitely thin screen ($t = 0$) that has been well studied to ([1]-[7]). As follows from Eqs.(17,18), to compute the values of coupling coefficients with an accuracy of up to a^3 the calculations of coefficients $\Lambda_{i,k}$ must be performed in the approximation $a = 0$. A question arises of how one should calculate coefficients $T^{(j)}_{m,s}$, since each term of the sum that determines these coefficients tends to zero at $a \to 0$. If this sum converges to any non-zero values, than one must take into account the infinite numbers of addends. This circumstance reflects the presence of singularity of the electromagnetic field on the aperture edges. This fact makes it practically impossible to employ the initial equations set (10) for calculating the characteristics of the coupled cavities in the case of small apertures. The modified set of equations (17,18) can be used to obtain solutions at small a-values with an accuracy of a^3. In the limit $a \to 0$ the contribution of addends with small l in the value of the sum, which determines $T^{(j)}_{m,s}$, tends to zero, while at large l, the difference of the adjacent Bessel function roots tends to the value π ($\lambda_{l+1} - \lambda_l \approx 0$), and the above sum tends to an integral independent of geometrical parameters

$$\lim_{a\to 0} \pi \frac{a}{b_j} \sum_{l=1}^{\infty} \frac{\theta_l^{(j)3} J_0^2(\theta_l^{(j)}) E_l\left(a/d_j, \nu_l^{(j)}\right)}{\chi_l \left(\lambda_m^2 - \theta_l^{(j)2}\right)\left(\lambda_s^2 - \theta_l^{(j)2}\right)} =$$

$$= \int_0^\infty \frac{J_0^2(\theta)\theta^2 d\theta}{\left(\lambda_m^2 - \theta^2\right)\left(\lambda_s^2 - \theta^2\right)} = K_{m,s}. \tag{22}$$

Since in the limit considered $f_m^{(i,j)} \to \lambda_m$, then from (20,21) it follows that $w_m^{(i,j)} \to w_m$, where w_m is the solution of the equation set

$$\sum_{s=1}^{\infty} K_{m,s} w_s = 3\pi / \left(2\lambda_m^2\right). \tag{23}$$

Coefficients $\Lambda_{i,k}$ do not depend on geometrical dimensions of cavities $\Lambda_{i,k} = \Lambda = \sum_s w_s/\lambda_s^2$. We have obtained the value of the constant Λ analytically: $\Lambda = 1$. Numerical calculations of the truncated set (20,21) also showed that with increasing S Λ tends to 1. This is in good agreement with the results of other authors (see [1]-[7]). Thus, at $S = 200$ and an appropriate choice of the step and the integrate interval in (22), when the solution of the set (23) becomes independent of these parameters, $\Lambda = 0,9989$.

As different from earlier approaches (see, for example, [1] - [4]), our equations set (20,21) permits to calculate the dependence of electromagnetic characteristics on any parameters, because no assumptions were made while obtaining it. On the base of such set one can easily calculate eigenfrequencies. We shall not dwell on it, but we shall perform analysis of the dependence of coefficients on such parameters that were impossible to study in earlier models.

First of all, let us become clear on the influence of electromagnetic field non-potentiality in the interaction region on $\Lambda_{i,k}$-values, since in all previous research studies ([1]-[4], [7]) on coupling through small-size holes the assumption about field potentiality in the vicinity of the hole were made. In our model investigation of this problem is reduced to studying

Table 1: Dependence of $\Lambda_{i,k} = \Lambda$ coefficients and coupling coefficients $\tilde{\Lambda}$ on frequency f ($d_1 = d_2 = 3.5\,\mathrm{cm}$, $b_1 = b_2 = 4\,\mathrm{cm}$, $t = 0.0\,\mathrm{cm}$, $f_{010} = 2.868563\,\mathrm{GHz}$)

	$a = 1\,\mathrm{cm}$		$a = 1.5\,\mathrm{cm}$	
f (GHz)	Λ	$\tilde{\Lambda}$	Λ	$\tilde{\Lambda}$
0	0.896590	0.012606	0.788984	0.037440
1	0.897783	0.012623	0.793784	0.037667
2	0.900862	0.012666	0.808207	0.038352
3	0.903614	0.012705	0.831250	0.039445

the dependence of the coefficients $\Lambda_{i,k}$ on the frequency f; the case $f = 0$ corresponds to the assumption of field potentiality in the interaction region.

Since frequency comes into the appropriate coefficients only in the form of expression $\Omega = \omega a/c$, then it follows that Λ-variation with increasing frequencies from 0 to f_{010} must be dependent on coupling aperture size - the smaller a the weaker dependence of $\Lambda_{i,k}$ on frequency. This is confirmed by the calculations results (Tab.1).

From Tab.1 it follows that an error in calculations of the coupling coefficients $\tilde{\Lambda}$ $\left(\tilde{\Lambda} = 2a^3\Lambda/\left(3\pi b_1^2 d_1 J_1^2\left(\lambda_1\right)\right)\right)$ at[5] $a = 1\,\mathrm{cm}$ is on the order of 10^{-4} (the equivalent frequency shift being $\approx 300\,\mathrm{kHz}$) and, consequently, all calculations can be made in static approximation. Yet, already for $a = 1.5\,\mathrm{cm}$ the error is of the order of $2\,10^{-3}$ (the equivalent frequency shift being $\approx 6\,\mathrm{MHz}$), which is inadmissible for precise calculations.

Of importance for applied use is the dependence of the coupled coefficients on the coupling aperture radius a. Analysis of the expression (19) indicates that $\Lambda_{i,k}$ depends both on the above parameter $\Omega = \omega a/c$ and on relation of a to all cavity geometrical parameters and screen thickness $(a/d_j, a/b_j, a/t, j = 1, 2)$. Results of the calculations of the relationship of interest on basic of our model in the static approach $(f = 0)$ are given in Tab.2.

Tab.2 also shows the results of calculations for various values of the parameter a/t at $a/d_j \to 0, a/d_j \to 0$ [9]. It follows from Tab.2 that taking into account the finitness of parameters a/d and a/b lead not only to a drastic change of the numerical values of the coupling coefficients, but to change the functional dependence of $\Lambda_{i,k}$ on a. For instance, at a finite thickness of the screen the coefficient $\Lambda_{1,1}$, which determines the cavity eigenfrequency shift, decreases with increasing a, contrary to what one can obtain from the results of the paper [9].

From the basic set of Eqs.(17,18) it follows that for identical cavities the dependence of the coupling coefficients on the cavity length d is determined by the ratio $\Lambda_{i,k}/d$. The presently developed models of coupled cavity model ([1]-[7]) are true when the condition $a/d \ll 1$ is satisfied. In this case $\Lambda_{i,k}$ are independent of d and coupling coefficients are inversely proportional to d.

Results of our calculations, as one can see from Tab.3, indicate that in the region

[5] Dimensions of the coupling hole of widely used disc-loaded waveguides at the operating frequency $f \approx 3\,\mathrm{GHz}$ are $a = 0.9 \div 1.5\,\mathrm{cm}$.

Table 2: Dependence of $\Lambda_{i,k}$ coefficients on the coupling aperture radius a ($d_1 = d_2 = 3.5\,\mathrm{cm}$, $b_1 = b_2 = 4\,\mathrm{cm}$, $f = 0$)

a, cm	$\Lambda_{i,k} = \Lambda$ ($t = 0$)	$\Lambda_{1,1}$ ($t = 0.4$) cm		$\Lambda_{1,2}$ ($t = 0.4$) cm	
			[9]		[9]
0.04	0.9976	0.8580	0.8584	0.0000	0.0000
0.10	0.9965	0.8571		0.0000	
0.133333	0.9956	0.8563	0.8584	0.0006	0.0006
0.20	0.9929	0.8539		0.0066	
0.30	0.9871	0.8489		0.0331	
0.40	0.9792	0.8422	0.8590	0.0734	0.0748
0.50	0.9695	0.8341		0.1181	
0.70	0.9448	0.8138		0.2016	
0.90	0.9140	0.7880		0.2675	
1.00	0.8965	0.7731		0.2934	
1.10	0.8777	0.7568		0.3150	
1.333333	0.8286	0.7139	0.8765	0.3500	0.4202
1.30	0.8360	0.7204		0.3462	
1.50	0.7889	0.6790		0.3633	

Table 3: Dependence of coefficients $\Lambda_{i,k}/d$ on the cavity lenght $d_1 = d_2 = d$ ($\Lambda_{i,k} = \Lambda$ — $t = 0$, $\Lambda_{1,1}, \Lambda_{1,2}$ — $t = 0.4\,\mathrm{cm}$, $b_1 = b_2 = 4\,\mathrm{cm}$, $f = 0$)

d, cm	$\Lambda_{i,k}/d = \Lambda/d$ ($t = 0$)	$\Lambda_{1,1}/d$ ($t = 0.4$) cm	$\Lambda_{1,2}/d$ ($t = 0.4$) cm
0.001	2.559185	5.071967	0.010858
0.050	2.434071	3.851450	0.357559
0.100	2.305363	3.198808	0.512277
0.200	2.062168	2.425438	0.598867
0.300	1.839241	1.952942	0.579913
0.500	1.460225	1.383953	0.478433
0.750	1.112953	0.992879	0.365433
1.000	0.877394	0.764395	0.287444
1.500	0.600848	0.516681	0.196466
2.000	0.451742	0.388012	0.147701
2.500	0.360623	0.310091	0.117952
3.000	0.299667	0.258049	0.098056
3.500	0.256169	0.220893	0.083854

$a/d > 1$ the dependence of coupling coefficients on d changes significantly; namely, at $d \to 0$ $\Lambda_{i,k}/d$ tend to constant values that are determined by other cavity geometrical dimensions. For instance, for the case $t \neq 0$ at $d \to 0$ the coefficient $\Lambda_{1,2}/d$, which determined the frequency difference between 0 and π-type eigenmodes of the system, tends to zero, while the coefficient $\Lambda_{1,1}/d$, determining the eigenfrequency shift, tends to non-zero value. This signifies that in the two-thin-cavity case ($d/t \ll 1$) difference between 0 and π type frequencies will be small. This result is in agreement with the circumstance that at $d = 0$ the two cavity system transforms into a single cavity with the radius $r = a$ and the length t. However, in this case one must takes into account the dependence of parameters on frequency.

5 Conclusion

On this way, we put forward a novel analytical model for studding the coupling of two cavities through an aperture in separating screen of finite thickness without making assumption on smallness of any parameters.

1. On the base of rigorous electromagnetic approach the coupling coefficients of the cylindrical cavities within the limit of small aperture and infinitely thin wall are calculated.

2. The presented numeric results of electromagnetic characteristic dependencies that have been impossible to perform on the base of previous models show that influence of non-potentiality of fields in the vicinity of the hole and nearness of cavity walls may be considerable.

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